Capture of K^- by the ^{4}He atom and the internal Auger effect in the $K\alpha e$ kaonic atom

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In this paper, we have investigated the interactions of K^- in a ^{4}He target using a quantum mechanical approach. For this purpose, we have used time-dependent perturbation theory and Fermi's golden rule to calculate the capture rate of K^- by the ^{4}He atom and the rate of internal Auger effect in the $K\alpha e$ atom. The initial distribution of n, ℓ states in the $K\alpha^+$ ion is also calculated. Some detailed analytical and numerical quantum mechanical calculations are performed for several transitions. Our results for the n, ℓ distributions in $K\alpha^+$ ions are necessary to begin a Monte Carlo simulation of the cascade processes in kaonic helium atoms.

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I. INTRODUCTION

The study of kaonic atoms is very important for investigating the K^--nucleus strong interaction at low energies. Several processes occur in the following sequences when the K^- enters a target: stopping of the kaon, kaonic atom formation in a highly excited state, and atomic cascade consisting of a multistep transition to lower atomic states. It is a complicated interplay of competitive collisions and radiative deexcitation processes. Radiative transitions produce x-rays. Strong interactions between the nucleus and kaon lead to a shift and finite absorption width for the x-ray series which can be measured [1–9].

Although the investigations of cascade processes of K^-p and K^-d atoms have already been done [9–16], the cascade dynamics for K^- ^{4}He atoms have not been fully understood yet. Fortunately, precise experiments have recently been carried out to detect x-ray yields of K^- ^{4}He atoms [3,5,9]. The kaon is captured in a highly excited atomic orbit, replacing one of the electrons which is ejected from the atom. Then, $K\alpha e$ atom is deexcited by internal Auger effect and the other electron is ejected. Subsequently, we have a $K\alpha^+$ ion which starts cascade processes until either the K^- is absorbed by the nucleus or decays. $K\alpha^+$ is deexcited to lower states by competitive cascade processes: Stark mixing, the external Auger effect, Coulomb transition, and radiative transitions [9–17].

In this paper we have calculated the interaction rates of the K^- in a ^{4}He target to determine the initial probability distribution of n, ℓ states in kaonic helium atom. It is necessary for the forthcoming Monte Carlo simulation of the cascade processes and x-ray yields of kaonic helium atoms. For this purpose, time-dependent perturbation theory is used in Sec. II to calculate the rate of K^- capture by ^{4}He or the $K\alpha e$ kaonic atom formation rate, and Fermi's golden rule is used in Sec. III to calculate the rate of the internal Auger effect in the $K\alpha e$ atom. It shoud be noted that our calculations are done without any free parameters.

For the quantum mechanical calculations, detailed analytical and numerical calculations should be done for many transitions. Then, the initial probability distributions for n, ℓ states in the $K\alpha e$ atom and $K\alpha^+$ ion are determined.

II. CALCULATION OF CAPTURE RATE OF A KAON BY A HELIUM ATOM

When the K^- enters a helium target, it loses its kinetic energy due to scattering, deexcitation, and ionization by helium atoms. The kinetic energy of the K^- decreases until the energy falls near the excitation energy of the first excited state of helium (19.8 eV); it has been shown that most of the kaons are captured before falling below this energy [17]. It is captured in a highly excited atomic orbit by replacing one of the electrons which is ejected from the atom. Capture of K^- by the helium atom is shown in Fig. 1.

If a semiclassical approach is used, $n \sim 25$ is calculated for the initial state of the K^- in the $K\alpha e$ atom. Indeed, we should use a full quantum mechanical approach in order to determine the initial distribution of n and ℓ for the K^- in the $K\alpha e$ atom. For this purpose, the Hamiltonian of the system (Fig. 1) is written as follows:

$$H = H_{\rm He} + \frac{p^2}{2\mu} - \frac{1}{r_{k\alpha}} + \frac{1}{2|\vec{r}_{k\alpha} - \vec{r}_1|} + \frac{1}{2|\vec{r}_{k\alpha} - \vec{r}_2|}, \quad (1)$$

where μ is the reduced mass of K^- and ^{4}He, p stands for the momentum operator of the kaon relative to the α particle and $H_{\rm He}$ is the Hamiltonian of the helium atom,

$$H_{\rm He} = \frac{p_1^2}{2} + \frac{p_2^2}{2} - \frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{2|\vec{r}_2 - \vec{r}_1|}, \quad (2)$$

where p_i is the momentum operator of the electrons relative to the alpha particle. We have used the units in Table I to write the Hamiltonians.

We consider the interaction terms of electrons with K^- as the perturbation potential,

$$H = H_0 + V e^{\eta t}, \quad 0 < \eta \ll 1, \quad (3)$$

where η is a small and positive parameter which is used for applying the time dependent perturbation theory, and the limit $\eta \to 0$ is taken at the end of calculation. $H_0 = H_{\rm He} + H_{k\alpha}$, $H_{k\alpha} = \frac{p^2}{2\mu} - \frac{1}{r_{k\alpha}}$, and $V = \frac{1}{2|\vec{r}_{k\alpha} - \vec{r}_1|} + \frac{1}{2|\vec{r}_{k\alpha} - \vec{r}_2|}$ is considered the perturbation potential.

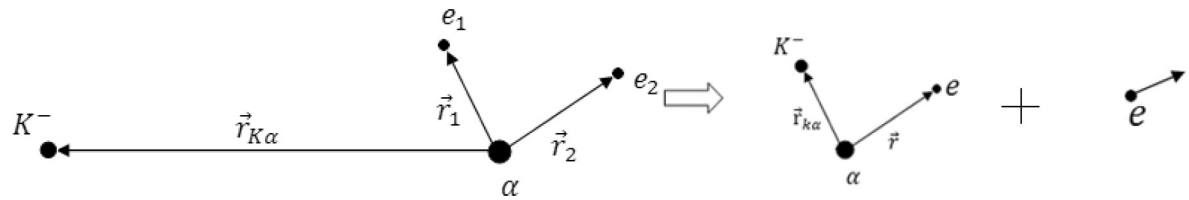

FIG. 1. Diagram of capture of K^- by the helium atom.

Before the collision, when the distance between the kaon and the helium atom is too large, the state ket in the interaction picture is assumed to be $|i\rangle$, which is the eigenket of the Hamiltonian H_0 with energy $E_i = \frac{q^2}{2\mu} + E_{0\mathrm{He}}$; q is the momentum of kaon relative to the α particle before collision and $E_{0\mathrm{He}}$ is eigenvalue of energy for the ground state of the helium atom. Therefore, the space part of the eigenket of the system before collision is $|i\rangle = |\psi^q_{K\alpha}, \psi^0_{\mathrm{He}}\rangle$, where $|\psi^0_{\mathrm{He}}\rangle$ is the ground state eigenket of the ^{4}He atom which is determined by variational method, and $|\psi^q_{K\alpha}\rangle$ is the eigenket of the incoming kaon with momentum q relative to the α particle, which can be determined using the hydrogen-like eigenket with positive energies:

$$\langle \vec{r}_{k\alpha}|\vec{q}\rangle = \frac{e^{i\vec{q}\cdot\vec{r}_{k\alpha}}}{L^{3/2}}\Gamma\left(1+\frac{i\mu_{k\alpha}}{q}\right)e^{\frac{\pi\mu_{k\alpha}}{2q}} \times {}_1F_1\left(-\frac{i\mu_{k\alpha}}{q}, 1; -i(qr_{k\alpha} + \vec{q}\cdot\vec{r}_{k\alpha})\right), \quad (4)$$

where $\mu_{k\alpha} = \frac{m_k m_\alpha}{m_k + m_\alpha}$ is the reduced mass of the kaon and α particle, ${}_1F_1$ is the hypergeometric function, and Γ is the gamma function.

After the collision of K^- with ^{4}He, we have a $K\alpha e$ atom and a free electron. The two electrons in the ^{4}He atom are identical fermions, so we have the space- symmetrical eigenket $|f\rangle = \frac{1}{\sqrt{2}}\{|\psi_{K\alpha e_1}, \psi_{e_2}\rangle + |\psi_{K\alpha e_2}, \psi_{e_1}\rangle\}$ after collision. Therefore, we need the eigenkets of the $K\alpha e$ atom ($|\psi_{K\alpha e}\rangle$) and the outgoing free electron in the Coulomb potential of the α particle ($|\psi_e\rangle$). To calculate $|\psi_{K\alpha e}\rangle$, we have used the time-independent perturbation theory:

$$H_{k\alpha e} = H_{k\alpha} + H_{e\alpha} + \frac{\lambda}{2|\vec{r}_{k\alpha} - \vec{r}|}, \quad (5)$$

where λ is a dimensionless perturbation parameter, and eventually we may set $\lambda \to 1$. $H_{k\alpha}$ and $H_{e\alpha}$ are the Hamiltonians for $K\alpha$ and $e\alpha$ systems, respectively:

$$H_{k\alpha} = \frac{p^2_{k\alpha}}{2\mu_{k\alpha}} - \frac{1}{r_{k\alpha}} \quad \text{and} \quad H_{e\alpha} = \frac{p^2_{e\alpha}}{2} - \frac{1}{r}, \quad (6)$$

TABLE I. Units used in the calculations. $\hbar$ is Planck's constant, m_e and e are the mass and charge of the electron, respectively, c is the speed of light, and α is the fine structure constant.

	Length	Time	Mass	Electric charge
Unit	$\frac{\hbar}{2m_e c\alpha}$	$\frac{\hbar}{m_e(2c\alpha)^2}$	m_e	$2e$

The hydrogen-like eigenkets are used for the bound eigenkets of $H_{k\alpha}$ and $H_{e\alpha}$,

$$H_{k\alpha}|n, \ell, m\rangle = \frac{-\mu_{k\alpha}}{2n^2}|n, \ell, m\rangle, \quad (7)$$

$$H_{e\alpha}|N, L, M\rangle = \frac{-1}{2N^2}|N, L, M\rangle, \quad (8)$$

where n, ℓ, m are the state quantum numbers of the $K\alpha$ atom and N, L, M are the state quantum numbers of the $e\alpha$ atom.

To calculate $|\psi_e\rangle$ for the ejected electron, we have used the hydrogen like eigenket with positive energies.

$$\langle \vec{\mathrm{r}}|\vec{k}\rangle = \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{\mathrm{r}}}\Gamma\left(1-\frac{i}{k}\right)e^{\frac{\pi}{2k}}{}_1F_1\left(\frac{i}{k}, 1; i(k\mathrm{r} - \vec{k}\cdot\vec{\mathrm{r}})\right), \quad (9)$$

where $\vec{k}$ is the momentum of the ejected electron relative to the $K\alpha e$ atom. It should be noted that the difference between the signs in Eqs. (4) and (9) is related to the opposite direction of the kaon and electron with respect to the α particle.

We have used time-dependent perturbation theory to determine the amplitude of the transition probabilities. For this purpose, many detailed calculations have been done. Then, the capture rate of a K^- by the ^{4}He atom is given by

$$\Gamma^{n\ell m}_C = \int N_{\mathrm{He}}\, V\, |V_{fi}|^2\, \frac{2\pi}{k}\delta(k - k')d^3k, \quad (10)$$

where the allowed value of $\vec{k}$ is $\vec{k}'$ which is determined by conservation of energy:

$$k' = \sqrt{\frac{q^2}{\mu} + 2E_{0\mathrm{He}} - 2E_{n\ell m}}. \quad (11)$$

Now we should determine the matrix elements V_{fi}. Because there is a singularity term $\frac{1}{|\vec{r}_{k\alpha}-\vec{r}|}$ in V_{fi}, it cannot be evaluated by a straightforward numerical method. To solve this problem, we have used Fourier transformation of $\frac{1}{|\vec{r}_{k\alpha}-\vec{r}|}$:

$$\frac{1}{|\vec{r}_{k\alpha} - \vec{r}|} = \int F(\vec{Q})\frac{e^{i\,\vec{Q}\cdot(\vec{r}_{k\alpha}-\vec{r})}}{(2\pi)^{3/2}}d^3Q. \quad (12)$$

It can be shown that $\frac{1}{|\vec{r}_{k\alpha}-\vec{r}|} = \frac{1}{2\pi^2}\int_0^\infty dQ \int_0^{2\pi} d\phi_Q \int_{-1}^{+1} e^{i\,\vec{Q}\cdot(\vec{r}_{k\alpha}-\vec{r})}d(\cos\theta_Q)$. After detailed calculations,

V_{fi} is given by:

$$V_{fi} = \frac{\sqrt{0.4893}\, i^{|m|}\left(\frac{|m|}{m}\right)^m \sqrt{\frac{(\ell+|m|)!}{(\ell-|m|)!}}}{\pi^2}(2z)^{\frac{3}{2}}\, e^{\frac{\pi}{2k}}\,\Gamma\left(1+\frac{i}{k}\right)\frac{2\pi e^{-im\phi_k}}{|m|!}\int_0^\infty (z^2+Q^2+k^2+2ikz)^{\frac{i}{k}}\,dQ$$
$$\times \int_{-1}^{+1} d(\cos\theta_Q)\int_0^\infty {r_{k\alpha}}^2 dr_{k\alpha}\int_{-1}^{+1} d\,(\cos\theta_{k\alpha})\left(\frac{-\varepsilon}{2}\right)^{|m|}\left(e^{iQr_{k\alpha}\cos\theta_{k\alpha}\cos\theta_Q}\langle\vec{r}_{k\alpha}|\vec{q}\rangle\langle\vec{r}_{k\alpha}|u_{n,l,0}\rangle\frac{p_\ell^{|m|}(\cos\theta_{k\alpha})}{p_\ell(\cos\theta_{k\alpha})}\right)$$
$$\times\left\{\frac{z\left(1+\frac{i}{k}\right)\left(2+\frac{i}{k}\right)_{|m|}}{a^{2(2+\frac{i}{k})}}\,{}_2F_1\left(\frac{\left(2+\frac{i}{k}+|m|\right)}{2},\frac{\left(2+\frac{i}{k}+1+|m|\right)}{2},|m|+1;\varepsilon^2\right)\right.$$
$$\left.-\frac{\frac{i}{k}(z+ik)\left(1+\frac{i}{k}\right)_{|m|}}{a^{2(1+\frac{i}{k})}(z^2+Q^2+k^2+2ikz)}\,{}_2F_1\left(\frac{\left(1+\frac{i}{k}+|m|\right)}{2},\frac{\left(1+\frac{i}{k}+1+|m|\right)}{2},|m|+1;\varepsilon^2\right)\right\}, \tag{13}$$

where $a^2 = z^2+Q^2+k^2+2\,Q\,k\,\cos\,\theta_Q\,\cos\,\theta_k$, $\varepsilon = \frac{a^2}{2\,Q\,k\,\sin\,\theta_k\,\sin\,\theta_Q}$, ${}_2F_1$ is the hyper- geometric function, Γ is the gamma function, and z is the effective nuclear charge.

Now we can calculate the probability distribution (P_n) for the n state, in which the K^- is captured and produces a $K\alpha e$ atom:

$$P_n = \frac{\Gamma_C^n}{\Gamma_C^{\rm tot}} = \frac{\sum_{\ell,m}\Gamma_C^{n\,\ell\,m}}{\sum_{n,\ell,m}\Gamma_C^{n\,\ell\,m}}, \tag{14}$$

where $\Gamma_C^{\rm tot}$ is the total capture rate of K^- by the ^{4}He atom.

Figure 2 shows P_n as a function of n states. It shows that the initial n states of a K^- in the $K\alpha e$ atom have a broad distribution around $n = 29$. Our calculations show that the results of a semiclassical approximation are not sufficient for the simulation of the cascade processes in the kaonic helium atom.

We have also compared our results for P_n with those based on the variational method to calculate the eigenkets for $K\alpha e$ atom [17,18]. Although Fig. 3 shows that they have similar results in some n states, we can see that they have also some differences. Calculations based on variational method [17,18] show that P_n has three peaks which are not reasonable, however our results have only a broad peak. Furthermore, our calculated P_n in highly excited states is less than the results from variational method. These differences can be explained as follows: The allowed ℓ states decrease if n is increased. Therefore, in high-n states the probability of the $K\alpha e$ atom formation has a decreasing behavior. Table II shows the maximum allowed ℓ state in some n states. When $n > 46$, there is no allowed ℓ. On the other hand, the cross section of the atomic formation in low-n states is smaller than those in highly excited n states.

III. CALCULATION OF THE RATE OF THE INTERNAL AUGER EFFECT IN THE $K\alpha e$ ATOM

When a K^- is captured by a ^{4}He atom, a highly excited $K\alpha e$ atom is produced. The $K\alpha e$ atom is a very unstable system and is deexcited by the internal Auger effect. The Auger electron carries the excitation energy and a $K\alpha^+$ ion is produced (Fig. 4).

By the same procedure used in previous sections, we can write Hamiltonian in the center-of-mass system:

$$H = H_{k\alpha} + H_{e\alpha} + \frac{1}{2|\vec{r}_{k\alpha}-\vec{r}|}, \tag{15}$$

where $H_{k\alpha} = \frac{p_k^2}{2\mu} - \frac{1}{r_{k\alpha}}$ and $H_{e\alpha} = \frac{p_e^2}{2} - \frac{1}{r_{e\alpha}}$.

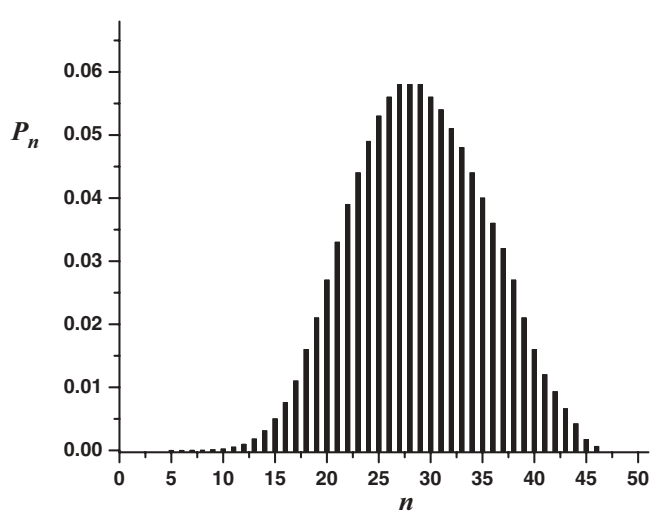

FIG. 2. Distribution of the n states at the instant of $K\alpha e$ atom formation (this work).

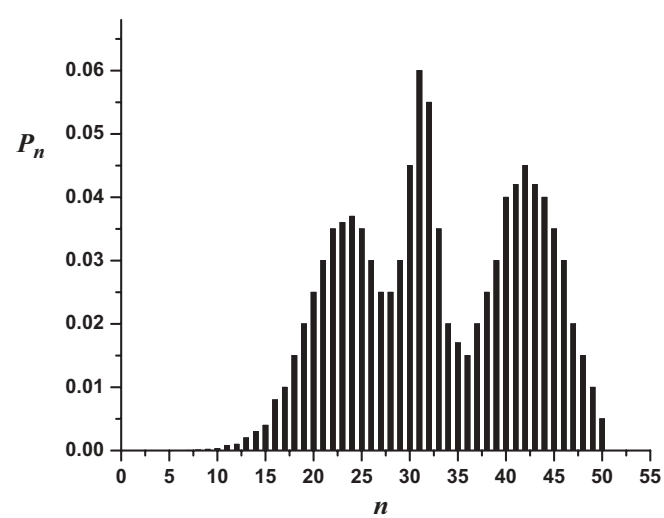

FIG. 3. Distribution of the n states at the instant of $K\alpha e$ atom formation using the variational method [17].

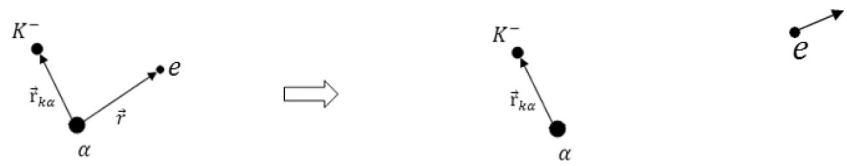

FIG. 4. Diagram of the internal Auger effect in $K\alpha e$ atom.

The third term of Eq. (15) is considered a perturbation potential, $V(t)$:

$$V(t) = \begin{cases} 0, & t < 0 \\ V = \frac{1}{2|\vec{r}_{k\alpha} - \vec{r}|}, & t > 0 \end{cases} \tag{16}$$

where $t = 0$ is the time of $K\alpha e$ production. Using Fermi's golden rule, we have

$$d\,\Gamma_{I.A.}^{(n\,\ell\,m\,|n'\ell'm')} = 2\pi\,|V_{fi}|^2\,\delta\,(E_f - E_i)\,d^3\,k = \frac{2\pi}{k}\,|V_{fi}|^2 \times \delta\,(k - k')\,d^3\,k, \tag{17}$$

where $E_{n\,\ell\,m}$ is the eigenvalue of the energy of the $K\alpha e$ atom. Because the probability of electron ejection is equal for all directions, the integral of Eq. (17) with respect to k is

$$\Gamma_{I.A.}^{(n\,\ell\,m\,|n'\ell'm')} = 8\pi^2 k'\,|V_{fi}|^2, \tag{18}$$

which is the internal Auger effect rate. Using the same procedure as in the previous section, we can determine the matrix elements V_{fi}. For this purpose, the z direction is selected in the same direction of $\vec{k}$. It can be shown that the V_{fi} is equal to zero unless $m' = m$. We can determine V_{fi} by detailed calculations as follows:

$$\begin{aligned} V_{fi} = {} & \delta_{m'}^{m}\frac{(2)^{\frac{3}{2}}e^{\frac{\pi}{2k}}\Gamma\left(1+\frac{i}{k}\right)}{\pi}\sqrt{\frac{(\ell-|m|)!\,(\ell'-|m|)!}{(\ell+|m|)!\,(\ell'+|m|)!}}\int_0^\infty dQ\int_{-1}^{+1}d\cos\theta_Q\int_0^\infty r_{k\alpha}^2 dr_{k\alpha}\int_{-1}^{+1}d\cos\theta_{k\alpha}\,J_0(Qr_{k\alpha}\sin\theta_Q\sin\theta_{k\alpha}) \\ & \times\left\{e^{iQr_{k\alpha}\cos\theta_{k\alpha}\cos\theta_Q}\langle\vec{r}_{k\alpha}|u_{n,\ell,0}\rangle\langle n',\ell',0|\vec{r}_{k\alpha}\rangle\frac{p_\ell^{|m|}(\cos\theta_{k\alpha})}{p_\ell(\cos\theta_{k\alpha})}\frac{p_{\ell'}^{|m|}(\cos\theta_{k\alpha})}{p_{\ell'}(\cos\theta_{k\alpha})}\frac{(1+Q^2+k^2+2ik)^{\frac{i}{k}}}{(1+|\vec{Q}+\vec{k}|^2)^{1+\frac{i}{k}}}\right. \\ & \left.\times\left[\frac{\left(1+\frac{i}{k}\right)}{1+|\vec{Q}+\vec{k}|^2}-\frac{\frac{i}{k}(1+ik)}{1+Q^2+k^2+2ik}\right]\right\} \end{aligned} \tag{19}$$

Now we can evaluate the $\Gamma_{I.A.}^{(n\,\ell m\,|n'\ell'm')}$ by Eq. (18). The total rate of the internal Auger effect in state n of a $K\alpha e$ atom is given by

$$\Gamma_{I.A.}^{n} = \sum_{\ell,m}\sum_{\substack{n',\ell',m' \\ n>n'}}\Gamma_{I.A.}^{(n\,\ell\,m\,|n'\ell'm')}. \tag{20}$$

It is evaluated numerically as shown in Fig. 5. We can see that the internal Auger effect is a very fast process and has a peak at $n = 20$.

The internal Auger effect rapidly leads to a new distribution of n', ℓ' states for the $K\alpha^+$ ion. The population of n', ℓ' for the $K\alpha^+$ ion due to the internal Auger effect is calculated as follows:

$$P_{n'\ell'} = \sum_{m'}\left\{\sum_{\substack{n\ell m \\ n>n'}}\frac{\Gamma_C^{n\ell m}}{\Gamma_C^{\text{tot}}}\times\frac{\Gamma_{I.A.}^{(n\ell m|n'\ell'm')}}{\Gamma_{I.A.}^{n\ell m}}\right\}, \tag{21}$$

TABLE II. The maximum allowed ℓ state in some n states.

n	38	39	40	41	42	43	44	45	46
ℓ	35	33	31	29	27	24	21	16	10

where $\Gamma_C^{\text{tot}} = \sum_{n\,\ell\,m}\Gamma_C^{n\,\ell\,m}$ and $\Gamma_{I.A.}^{n\ell m} = \sum_{n'\ell'm'}\Gamma_{I.A.}^{(n\ell m|n'\ell'm')}$. $\frac{\Gamma_C^{n\ell m}}{\Gamma_C^{\text{tot}}}$ is the probability of the $K\alpha e$ atom formation in an n, ℓ, m state and $\frac{\Gamma_{I.A}^{(n\ell m|n'\ell'm')}}{\Gamma_{I.A}^{n\ell m}}$ is the probability of the $K\alpha^+$ production in an n', ℓ', m' state due to the internal Auger effect. Therefore, when the $K\alpha^+$ ion is produced, the initial distribution for the

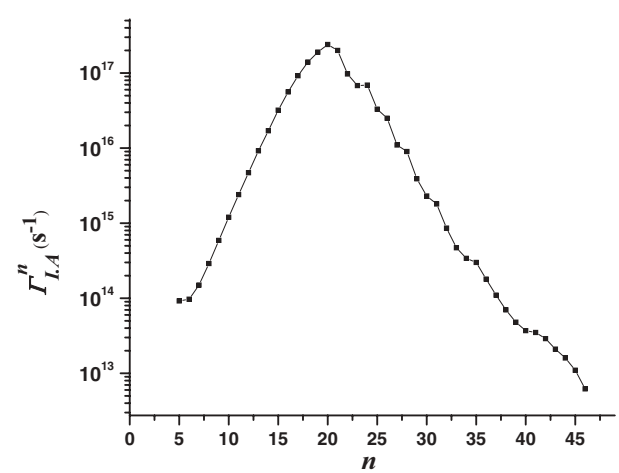

FIG. 5. Internal Auger rate as a function of orbital n state of the $K\alpha e$ atom.

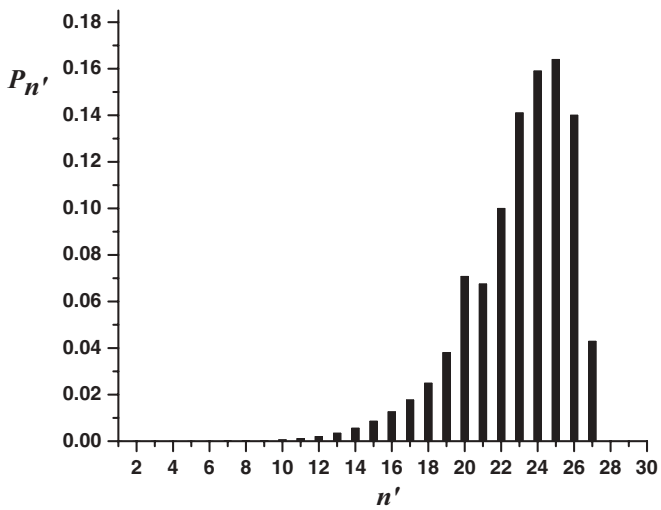

FIG. 6. Initial distribution of the orbital n state in the $K\alpha^+$ ion, before cascade processes.

n' state is given by

$$P_{n'} = \sum_{\ell'=0}^{n'-1} P_{n',\ell'} \tag{22}$$

which is shown in Fig. 6. Figure 6 shows that the maximum allowed value of the n' state is 27, when a $K\alpha^+$ ion is produced. In addition, the population of the $K\alpha^+$ ion has a peak at $n' = 25$. It shows that 16.4% of $K\alpha^+$ ions is produced in the $n' = 25$ state.

IV. CONCLUSIONS

In this paper, we investigated the interactions of a K^- in a ^{4}He target. Capture rates of the K^- by ^{4}He atoms and the internal Auger effect in the $K\alpha e$ atom were calculated by a quantum mechanical approach. For this purpose, we have used time-dependent perturbation theory and Fermi's golden rule. Many detailed analytical and numerical calculations were done in order to calculate the rates of the mentioned processes.

These calculations are necessary for a Monte Carlo simulation of the cascade processes of kaonic helium atoms. As we discussed in this paper, after the K^- beam enters a ^{4}He target, eventually $K\alpha^+$ ions are produced and actually the cascade processes takes place in $K\alpha^+$ ions. So, to begin the simulation of the cascade processes, initial n and ℓ states in the $K\alpha^+$ ion should be determined as performed in this paper.

Furthermore, our quantum mechanical approach can be used to calculate the cascade processes rates in $K\alpha^+$ ions. We follow the same procedure mentioned in this paper to evaluate the cascade rates of Stark mixing, the external Auger effect, Coulomb deexcitation, and radiative transitions in $K\alpha^+$ ions. We will use the results in forthcoming Monte Carlo simulation of the cascade processes in $K\alpha^+$ ions.

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